APPLICATION OF THE HARMONIC-BALANCE METHOD TO THE STABILITY ANALYSIS OF OSCILLATORS

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ABSTRACT

This paper outlines the application of harmonic-balance stability analysis to the design of oscillator circuits. Small-signal stability criteria and large-signal operation are considered. An improved picture of the stable regions of circuit operation and determination of physically stable operating points is demonstrated.

INTRODUCTION

Analysis of stability using the invariant K and B1 factors has been a standard method of microwave circuit design since reported in [1] . However, K and B1 are based on finding impedances in the source or load plane which result in reflection coefficients greater than one on the opposite plane at a given frequency. Information of the potential instabilities at other frequencies, as well as a more rigorous system analysis, are lacking in the K and B1 factors. Instead, a small-signal method is pursued here to determine the existence of the right-hand plane (RHP) zeros of the system using Nyquist's analysis [2].

The existence of RHP zeros indicates that a small perturbation of the system will result in a permanent deviation with an asynchronous transient (an abrupt frequency change will take place). This behavior indicates that the circuit has the potential to oscillate and one of the detected zeros in the RHP will likely determine the approximate frequency of oscillation. This analysis provides detection of asynchronous stability in the DC sense (termed DC Nyquist Analysis) since the circuit is linearized about its DC bias point with all RF sources killed.

Once the detection of RHP zeros is accomplished, there remains the question of the large-signal solution of the circuit. Microwave oscillator circuits using broadband active devices with broadband feedback circuits often exhibit two or more resonant frequencies where oscillation may take place due to the periodic nature of distributed resonators. Harmonic-balance simulation yields results only for the simulation frequency selected, and it will be shown that harmonic-balance analysis can produce several oscillatory solutions. A method is needed to determine which of these solutions are physically realizable as opposed to numerically simulated. A method based on [3] has been incorporated into Microwave Harmonica [4] for the detection of bifurcations in the parameterized sweep analysis of an arbitrary circuit. This analysis determines the synchronous instability of a system. Synchronous instability indicates that a small perturbation of the system will result in a permanent deviation with a synchronous transient (no abrupt frequency changes will take place). The information derived from a synchronous analysis include:

- the point at which an oscillator starts (termed a Hopf bifurcation)
- oscillator drop-outs, e.g. as in a VCO
- hysteresis critical points (termed turning points)

The next two sections of the paper describe the DC Nyquist analysis and application examples. The two following sections discuss the synchronous stability analysis and application examples.

DC NYQUIST ANALYSIS

Formal Nyquist analysis performs an integration of the system characteristic equation in the complex plane as shown in Figure 1. The path of integration begins at $0^+-j\infty$, proceeds to $0^++j\infty$ (avoiding any zeros on the $j\omega$ axis), and follows a semicircle of infinite radius back to $0^+-j\infty$. If there are natural frequencies in the RHP, these will be enclosed by the path of integration, while those in the LHP will not. By observing the result of the integration, it can be determined if there are natural frequencies in the RHP and therefore determine whether the circuit is asynchronously unstable.

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The implementation of Nyquist's analysis does not perform the integration explicitly, but provides similar information. The determinant of the closed-loop system under study is computed from a low frequency to a high frequency and plotted on the complex plane. If the path crosses the negative real axis and encircles the origin, then a natural frequency (σ+jω) exists in the right-hand plane and the system is asynchronously unstable. This is shown in Figure 2 for the cases where $\sigma < 0$ (corresponding to a zero in the LHP) and $\sigma > 0$ (corresponding to a zero in the RHP).

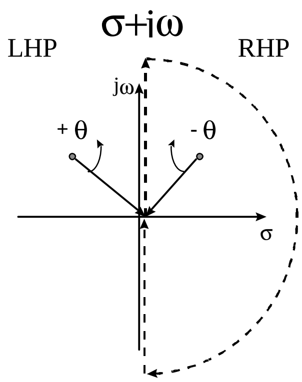

Figure 1: Nyquist integration path

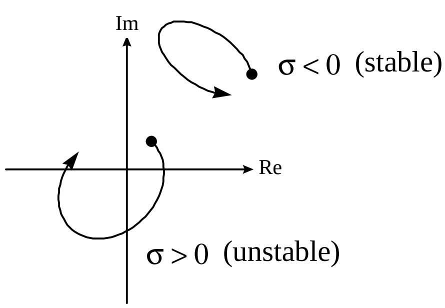

Figure 2: Plot of the system determinant for a stable ($\sigma < 0$) and unstable ($\sigma > 0$) case.

The frequency sweep should be performed to a high enough frequency where it is known that the circuit will not exhibit any resonant features or to the f_{max} of the active device. A very wide frequency range, often nine or ten decades, is used to capture low frequency resonances due to bias circuitry and high frequency resonances due to parasitics and periodic structures. Clearly a simple regular step sweep is not appropriate. Circuit resonances are often of sufficient Q that a regular-step sweep can easily miss the resonant frequency entirely unless very fine sweep points are selected or the frequency of the resonance is known a priori. To alleviate these problems, an adaptive frequency sweep which monitors the rate of change of the determinant and adjusts the frequency delta accordingly is used. The analysis is more efficient than a simple linear or logarithmic sweep.

APPLICATION OF DC NYQUIST ANALYSIS

An example application is demonstrated using a MESFET frequency doubler shown in Figure 3. The Nyquist plot is shown in Figure 4 where the frequency sweep was from 1Hz to 20GHz. As seen, the trace does not cross the negative real axis indicating there are no natural frequencies in the RHP and the circuit is stable ($\sigma < 0$).

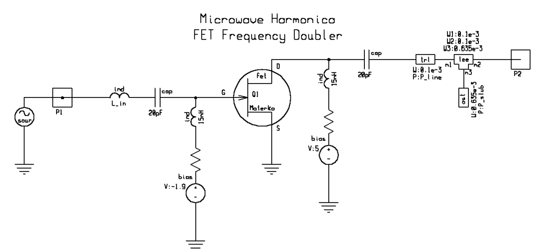

Figure 3: 5GHz MESFET frequency doubler.

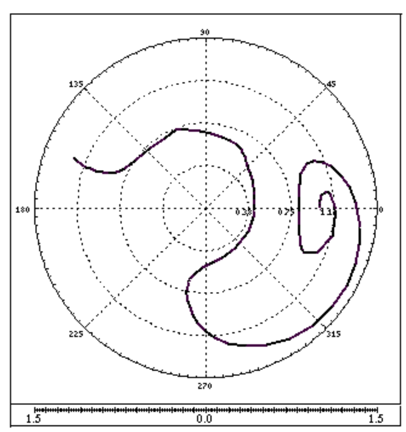

Figure 4: Nyquist plot of the frequency doubler in the range of 1 Hz to 20 GHz. Asynchronous stability is indicated since the path does not cross the negative real axis and it does not encircle the origin.

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To demonstrate asynchronous instability, a MESFET oscillator shown in Figure 5 is investigated. It is known by design that this circuit has a natural frequency in the RHP near 5GHz and is therefore asynchronously unstable. The Nyquist plot from 1 Hz to 20 GHz is shown in Figure 6. Since the path crosses the negative real axis, the analysis has indicated asynchronous instability. It is interesting to note that it crosses the axis three times possibly indicating three natural frequencies. We will later show the analysis of these three points.

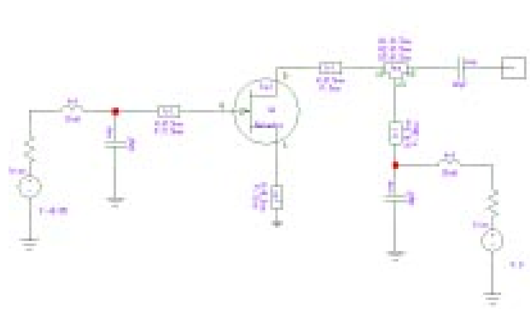


Figure 5: 5GHz MESFET oscillator.

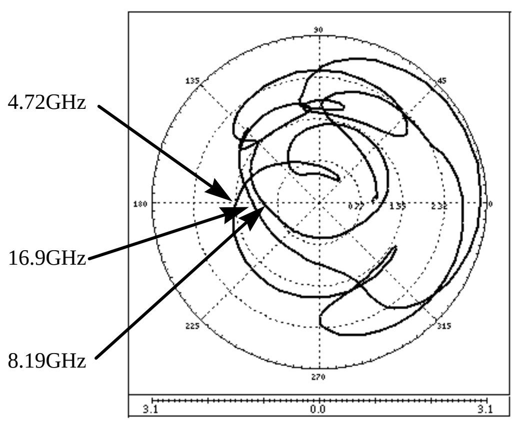


Figure 6: Nyquist plot of the oscillator.

SYNCHRONOUS STABILITY ANALYSIS

Synchronous stability analysis uses harmonic-balance analysis to trace the solution path to determine bifurcations of a circuit operating condition. Mathematically, a bifurcation takes place when the zeros of a circuit exchange sign on their real parts, or when the solution path splits into two or more distinct curves. From a circuit designer's point-of-view, these bifurcations are characterized by the following:

- Change from a stable DC operating point to a oscillatory regime
- Hysteresis in a physically observable circuit response
- Physical observation of a subset of computed operating regimes

Two important concepts in the determination of synchronous stability are derived from differential equation theory. The first is simply stated [5]:

> The stability of two points on the solution path curve of a nonlinear function are the same if there is no bifurcation point between the two points.

The second concept helps determine the stability when crossing a bifurcation point and can be summarized as follows [5]:

> Stability is exchanged at a turning point bifurcation.
> Stability is maintained at a critical point on the new solution path in the same direction of the parameter.

Therefore, if we can absolutely determine the synchronous stability of one point on the solution path, then we can determine the stability of any point on the path if the bifurcations are known. From the circuits point-of-view, consider Figure 7 where the circuit response P_{out} is plotted as a function of a bias source E in the absence of any RF excitation. From obvious physical considerations, point A is physically stable because it is at rest without any excitation and no observable output. Point H1 is a Hopf bifurcation point. Points B and C are turning points.

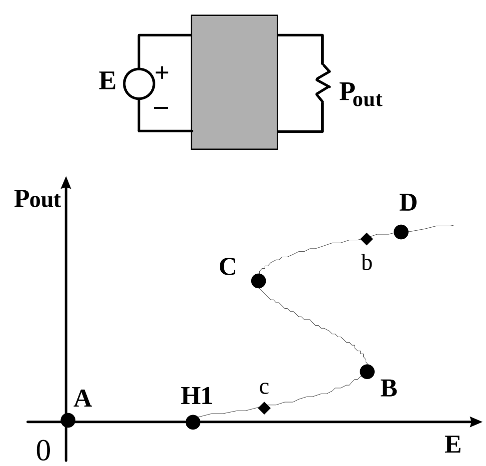


Figure 7: Consideration of a circuit response P_{out} as a function of applied DC bias E.

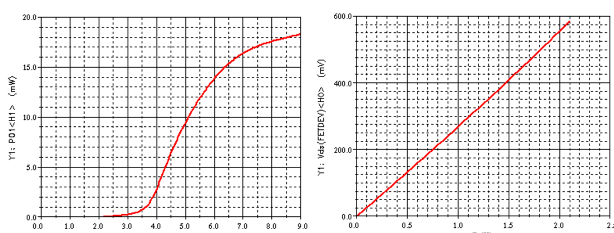
When a circuit exhibits a characteristic as shown in Figure 7, the physically observable behavior does not include the unstable path BC. A hysteresis curve is then observed where the output jumps from B to b for increasing E and from C to c for decreasing E.

APPLICATION OF SYNCHRONOUS STABILITY

An example application demonstrating synchronous stability is a MESFET oscillator as shown previously in Figure 5. Harmonic-balance oscillator analysis finds an oscillatory regime near 5 GHz at the designed bias point of Vds = 9 V. Using the solution path analysis we can determine the synchronous stability of the oscillator in two steps:

Step 1	Sweep bias from 9V until the bifurcation point is reached where oscillation ceases. This produces a Hopf bifurcation point.
Step 2	Sweep the bias from the end point of Step 1 until 0V where a known physically stable point is reached.

The result of the analysis is shown in Figure 8. Figure 8A shows the AC output power of the oscillator as the bias is swept and it undergoes a Hopf bifurcation at 2.2V. The bias is then swept down to 0V in Figure 8B and an observable DC output such as Vds is plotted. At 0V we know that the system is stable, so a conclusion can be drawn that the oscillating solution is stable since the path of Figure 8A is in the same direction as the parametric bias.



A B

Figure 8: Two-step process to determine synchronous stability of the MESFET oscillator.

When asynchronous stability was discussed for the MESFET oscillator in Figure 6, it was noted that three possible unstable frequencies may exist due to the Nyquist plot crossing the negative real axis three times. We can investigate this behavior to fully determine the stable operating region of this oscillator. The three resonant frequencies are estimated at the frequency where the Nyquist trajectory crosses the negative real axis and using oscillator analysis, we can obtain Figure 9 showing what appear to be three oscillatory regimes.

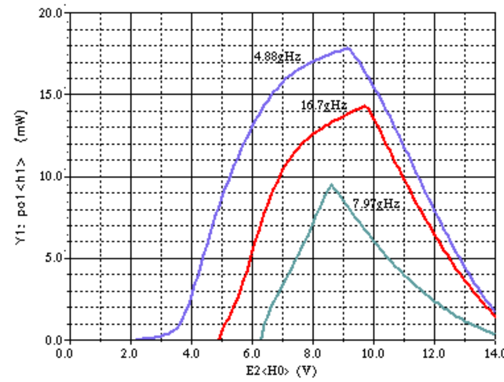


Figure 9: Three oscillatory regimes for the oscillator.

These three solution paths are all synchronously stable, however, we know that only one solution can physically exist. The answer lies in the observation that the solution path at 4.88GHz (top line) has the first Hopf bifurcation point at 2.2V. When the bifurcation for the 16.7GHz solution path or the 7.97GHz solution path occurs at 4.9V and 6.2V respectively, we are already operating in the stable 4.88GHz regime. The introduction of these other Hopf bifurcations will not modify the state of the first. Therefore, the natural frequencies introduced by the 16.7GHz and 7.97GHz regimes are asynchronously unstable and are not physically observable.

ACKNOWLEDGEMENT

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